Question 4 from the book: What are the coordinates of the vector *0* in the Euclidean plane that satisfies property 3 on page 3? Justify your answer.

Xingjian's solution

Let **0** denote the zero vector in the Euclidean plane. By definition, the zero vector must satisfy Property 3:

$$\mathbf{x} + \mathbf{0} = \mathbf{x} \quad \text{for all vectors } \mathbf{x}.$$

Let $\mathbf{x}$ be any vector with coordinates (a, b), and suppose the zero vector has coordinates (u, v). Using componentwise vector addition in $\mathbb{R}^2$, we have

$$(a, b) + (u, v) = (a + u,\ b + v).$$

By Property 3, this sum must equal (a, b). Hence,

$$(a + u,\ b + v) = (a, b).$$

Equating corresponding components gives

$$u = 0, \qquad v = 0.$$

Therefore, the coordinates of the zero vector in the Euclidean plane are

$$\mathbf{0} = (0, 0).$$

□

Question 5 from the book: Prove that if the vector x emanates from the origin of the Euclidean plane and terminates at the point with coordinates a_1, a_2, then the vector tx that emanates from the origin terminates at the point with coordinates ta_1, ta_2.

Xingjian's solution The vector x emanating from the origin and terminating at (a_1, a_2) can be written as

$$x = \langle a_1, a_2 \rangle.$$

By definition of scalar multiplication of vectors,

$$tx = t\langle a_1, a_2 \rangle = \langle ta_1, ta_2 \rangle.$$

The vector $\langle ta_1, ta_2 \rangle$ emanates from the origin and terminates at the point (ta_1, ta_2). Therefore, the vector tx terminates at (ta_1, ta_2). □

Question 6 from the book: Show that the midpoint of the line segment joining the points (a, b) and (c, d) is $(a + c)/2, (b + d)/2$.

Xingjian's solution

Let $A = (a, b)$ and $B = (c, d)$, and let $M = (x, y)$ be the midpoint of the line segment joining A and B. By definition of midpoint,

$$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}.$$

Using coordinates,

$$\overrightarrow{AB} = (c-a,\, d-b), \qquad \overrightarrow{AM} = (x-a,\, y-b).$$

Thus,

$$(x-a,\, y-b) = \frac{1}{2}(c-a,\, d-b) = \left(\frac{c-a}{2},\, \frac{d-b}{2}\right).$$

Equating components yields

$$x = \frac{a+c}{2}, \qquad y = \frac{b+d}{2}.$$

Therefore, the midpoint of the line segment joining (a, b) and (c, d) is

$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right). \square$$

Question 7 from the book: Prove that the diagonals of a parallelogram bisect each other.

Xingjian's solution Let $ABCD$ be a parallelogram in the Euclidean plane. Assume the position vectors of points A, B, D (with respect to the origin) are

$$\mathbf{a},\ \mathbf{b},\ \mathbf{d},$$

respectively. Since $ABCD$ is a parallelogram, the fourth vertex C satisfies

$$\mathbf{c} = \mathbf{b} + \mathbf{d} - \mathbf{a}.$$

Consider the diagonals AC and BD.

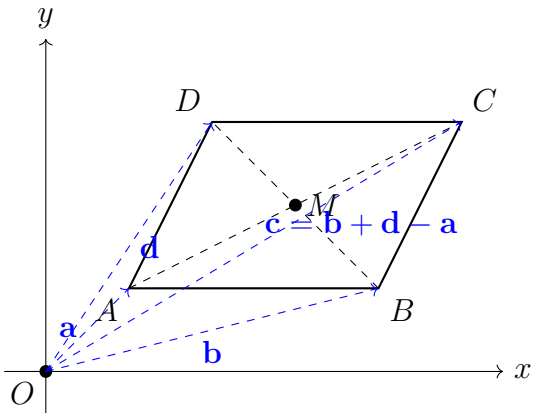
The midpoint of diagonal AC has position vector

$$\frac{\mathbf{a}+\mathbf{c}}{2} = \frac{\mathbf{a}+(\mathbf{b}+\mathbf{d}-\mathbf{a})}{2} = \frac{\mathbf{b}+\mathbf{d}}{2}.$$

The midpoint of diagonal BD has position vector

$$\frac{\mathbf{b}+\mathbf{d}}{2}.$$

Since both diagonals have the same midpoint, the diagonals intersect at this point and bisect each other. Therefore, the diagonals of a parallelogram bisect each other. $\square$



MAS 4105 Questions for Section 1.1

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